

Units and circuit variables

$T = 10^{12}$, $G = 10^9$, $M = 10^6$, $k = 10^3$, $m = 10^{-3}$, $\mu = 10^{-6}$, $n = 10^{-9}$, $p = 10^{-12}$. Convert first.

$$i = \frac{dq}{dt}, \quad q(t) - q(t_0) = \int_{t_0}^t i(\tau) d\tau, \quad v = \frac{dw}{dq}$$

$1 \text{ A} = 1 \text{ C/s}$, $1 \text{ V} = 1 \text{ J/C}$, $1 \text{ W} = 1 \text{ J/s}$; $1 \text{ kWh} = 3.6 \text{ MJ}$.
 $p = dw/dt$ and $\Delta w = \int p(t) dt$ (constant p : $\Delta w = p\Delta t$).

Power and passive signs

If the current reference enters labeled $+$, $p_{\text{abs}} = vi$. If it enters labeled $-$, $p_{\text{abs}} = -vi$. $p_{\text{abs}} > 0$: absorbs; $p_{\text{abs}} < 0$: supplies.

$$p_R = i^2 R = \frac{v^2}{R} \geq 0, \quad \sum p_{\text{abs}} = 0$$

Example: 12 V , 2 A entering the $-$ terminal gives $p_{\text{abs}} = -(12)(2) = -24 \text{ W}$ (supplies 24 W). Negative v or i means opposite actual polarity or direction.

Elements, Ohm's law, KCL/KVL

Ideal voltage source fixes v ; ideal current source fixes i . Dependent source: add its control equation, e.g. $v_s = \mu v_x$ or $i_s = \beta i_x$. Unequal ideal voltage sources in parallel or unequal ideal current sources in series conflict.

$$v = iR \text{ (current enters +)}, \quad G = \frac{1}{R}, \quad i = Gv$$

If current enters $-$, $v = -iR$. Open: $i = 0$; short: $v = 0$.

$$\text{KCL: } \sum i_{\text{in}} = \sum i_{\text{out}}, \quad \text{KVL: } \sum v_{\text{rise}} = \sum v_{\text{drop}}$$

Move $- \rightarrow +$ for a rise, $+ \rightarrow -$ for a drop. Keep directions consistent. Same node means continuous wire; crossings require a dot.

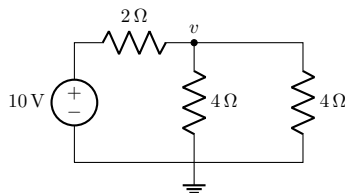
Series and parallel

Series: shared node has no other branch $\Rightarrow$ same current. Parallel: both endpoints share the same two nodes $\Rightarrow$ same voltage.

$$R_s = \sum R_j, \quad \frac{1}{R_p} = \sum \frac{1}{R_j}, \quad R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

For positive R : series equivalent exceeds each member; parallel equivalent is below the smallest member. Reduce, then work backward.

Worked KCL: solve a node voltage



1. Bottom wire is 0 V ; top of source is 10 V .
2. Current into node equals currents out:

$$\frac{10 - v}{2} = \frac{v}{4} + \frac{v}{4}.$$

3. Solve $v = 5 \text{ V}$. Check: $2.5 = 1.25 + 1.25 \text{ A}$.

Voltage and current division

Series group with total drop v ; parallel group with entering current i :

$$v_j = v \frac{R_j}{R_{\text{eq}}}, \quad i_j = i \frac{R_{\text{eq}}}{R_j}$$

Two resistors:

$$v_1 = v_s \frac{R_1}{R_1 + R_2}, \quad i_1 = i_s \frac{R_2}{R_1 + R_2}$$

Larger series R gets more voltage; smaller parallel R gets more current. **Loaded divider:** combine the load with the output resistor first:

$$v_{\text{out}} = v_s \frac{R_2 \parallel R_L}{R_1 + (R_2 \parallel R_L)}$$

Open load gives no-load value. Shorted load gives $v_{\text{out}} = 0$.

Fast solve path

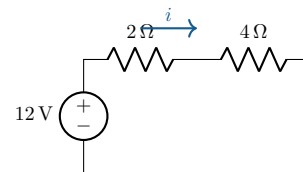
1. Mark nodes, current arrows, and voltage polarities; convert units.
2. Identify true series/parallel groups; reduce or write KCL/KVL.
3. Solve source current/voltage; work backward for internal values.
4. Find requested power using the appropriate terminal references.
5. Check KCL, KVL, $\sum p_{\text{abs}} = 0$, units, and $p_R \geq 0$.

Example: 12 V across $2 \Omega + (6 \Omega \parallel 3 \Omega)$. The parallel part is 2Ω , total 4Ω , source current 3 A . Parallel-group voltage is 6 V ; branch currents are 1 A and 2 A .

Useful resistor relations

For a rated resistive load, $R = V^2/P = P/I^2$. For parallel branches, first find $v = i_{\text{total}} R_{\text{eq}}$, then $i_j = v/R_j$. For series, find $i = v_{\text{total}}/R_{\text{eq}}$, then $v_j = iR_j$. **Don't mix up:** voltage-divider numerator = target R ; two-way current-divider numerator = the other R .

Worked KVL: solve a loop current



1. Choose clockwise i . Source is a 12 V rise; resistors drop $2i$ and $4i$.
2. Traverse clockwise: $-12 + 2i + 4i = 0$.
3. $i = 2 \text{ A}$; drops $4 + 8 = 12 \text{ V}$.

Meters and loading

Ammeter in series: ideal $R_A = 0$ (short); real $R_A > 0$ adds series resistance. Voltmeter in parallel: ideal $R_V \rightarrow \infty$ (open); real R_V shunts the measured branch. **Insert the meter equivalent resistance and re-solve** to find the actual reading.

d'Arsonval meter design

Movement full-scale V_m, I_m has coil $R_m = V_m/I_m$. For desired ammeter full scale I_{FS} , add a parallel shunt:

$$R_{sh} = \frac{V_m}{I_{FS} - I_m}, \quad R_{A,eq} = \frac{V_m}{I_{FS}}$$

For desired voltmeter full scale V_{FS} , add a series resistor:

$$R_s = \frac{V_{FS} - V_m}{I_m}, \quad R_{V,eq} = \frac{V_{FS}}{I_m}$$

Example: 50 mV, 1 mA movement $\Rightarrow R_m = 50 \Omega$. For a 10 mA ammeter, $R_{sh} = 5.56 \Omega$ and $R_{A,eq} = 5 \Omega$. For a 150 V voltmeter, $R_s = 149,950 \Omega$ and $R_{V,eq} = 150 \text{ k}\Omega$.

Wheatstone bridge

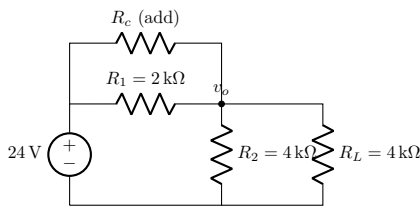
Balance: detector current $i_g = 0$ and midpoint potentials are equal. For textbook Fig. 3.28 labels:

$$R_x = \frac{R_2}{R_1} R_3$$

If labels differ, equate the two midpoint divider voltages and solve. At balance, each side's branch pair can be treated as series.

Worked design: add a load, choose R_c

An original 24 V divider has $R_1 = 2 \text{ k}\Omega$ and $R_2 = 4 \text{ k}\Omega$, giving $v_o = 16 \text{ V}$. Add a $4 \text{ k}\Omega$ load R_L . Put a new resistor R_c across R_1 to keep $v_o = 16 \text{ V}$.



1. New lower equivalent: $R_d = R_2 \parallel R_L = 2 \text{ k}\Omega$.
2. Required upper equivalent from divider rule:

$$16 = 24 \frac{2}{R_u + 2} \quad \Rightarrow \quad R_u = 1 \text{ k}\Omega.$$

3. Since $R_u = R_1 \parallel R_c$, $1 = 2R_c/(2+R_c)$ in $\text{k}\Omega$, so $R_c = 2 \text{ k}\Omega$. Check: without R_c , added load gives only 12 V.

Δ -Y resistor conversion

Triangle nodes a, b, c have edges R_{ab}, R_{bc}, R_{ca} . Y arms R_a, R_b, R_c meet at a center and connect to matching outer nodes. Let $S = R_{ab} + R_{bc} + R_{ca}$.

$$R_a = \frac{R_{ab}R_{ca}}{S}, \quad R_b = \frac{R_{ab}R_{bc}}{S}, \quad R_c = \frac{R_{bc}R_{ca}}{S}$$

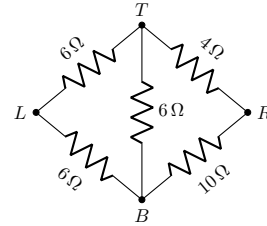
For Y to Δ , let $T = R_aR_b + R_bR_c + R_cR_a$:

$$R_{ab} = \frac{T}{R_c}, \quad R_{bc} = \frac{T}{R_a}, \quad R_{ca} = \frac{T}{R_b}$$

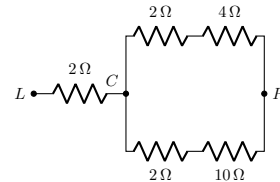
Equal Δ edges $R \rightarrow$ Y arms $R/3$; equal Y arms $R \rightarrow \Delta$ edges $3R$. Use a transform when no simple series/parallel group exists.

Worked bridge $\rightarrow$ Y

Across $L-R$, the left triangle $L-T-B$ has three 6Ω edges. Its Y arms are all $6/3 = 2 \Omega$.



After replacing the left triangle (center C):



1. Replace only the $L-T-B$ triangle by a Y with center C : $R_{LC} = R_{CT} = R_{CB} = 2 \Omega$.

2. From C to R , the upper route is $2 + 4 = 6 \Omega$; lower route is $2 + 10 = 12 \Omega$.

3. Routes are parallel, then add the left arm:

$$R_{LR} = 2 + (6 \parallel 12) = 2 + 4 = \boxed{6 \Omega}.$$

For 12 V across $L-R$, source current is $12/6 = 2 \text{ A}$.

Exam traps and checks

- Sharing one node is not enough for parallel. A shared node with a third branch prevents a series pair.
- A negative solved current means flow opposite the arrow; a negative voltage means reversed polarity. Neither is automatically an error.
- Equivalent R preserves terminal behavior only. Reverse the reduction to get a particular internal branch value.
- A finite-resistance meter changes the network. Find its reading *after* inserting it into the model.
- Verify a branching-node KCL, a loop KVL, $\sum p_{abs} = 0$, units, equivalent-resistance bounds, and $p_R \geq 0$.